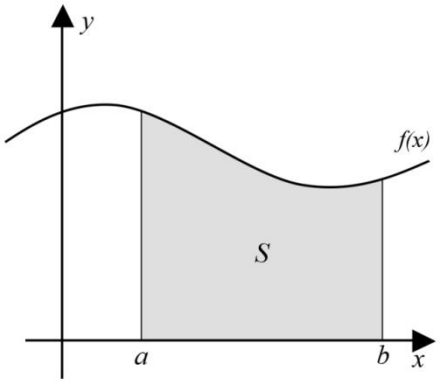
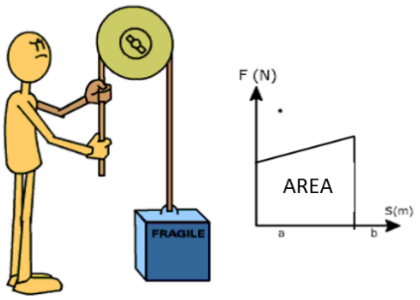
Integration – Finding Area Under Functions

Besides finding the slopes or "steepness" of curves, Calculus also helps us solve for the ***AREA*** under a curve. Finding the AREA under a function can be very useful in science and mathematics. What the AREA actually represents, depends on the function, but it is often a very practical quantity to find. The technique in Calculus used to find the area under curves is called: ***Integration***



Whys is Area so important?

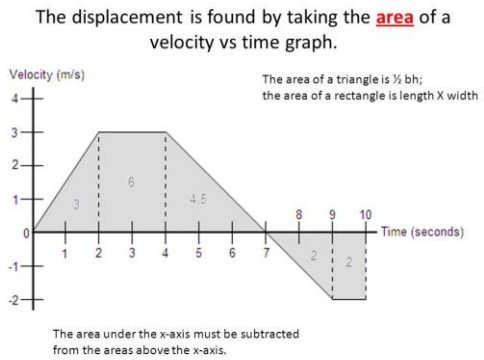
Again, depending on the function and the situation, area under a graph can have a **practical meaning other than just area**. Here are some examples:



Examples:

1. The AREA under a Force vs. Distance graph in physics is used to calculate ***WORK*** or ***ENERGY***.

2. We can use integration to find displacement, and Velocity.

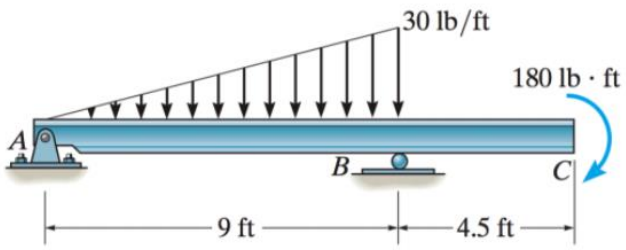


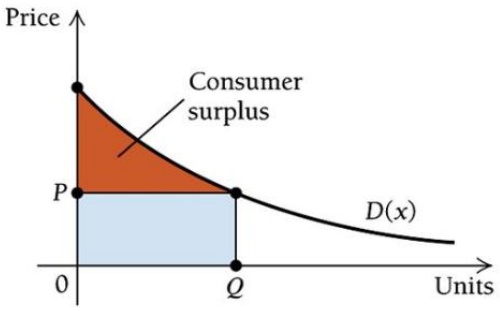
3. Volumes

4. Economic Values (Consumer Surplus)

5. Center of Mass

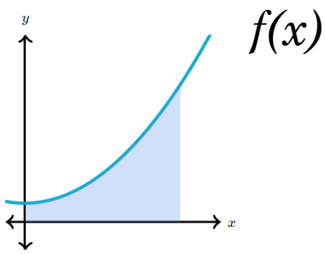
6. Force distribution



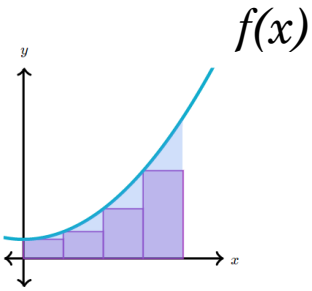


The essence of an **Integral** (adding up rectangles….lots of rectangles).

Suppose we want to find the area under this curve:



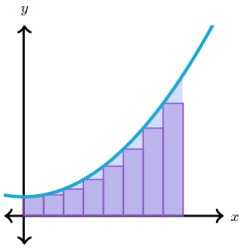
We may struggle to find the exact area, but we can approximate it using rectangles:

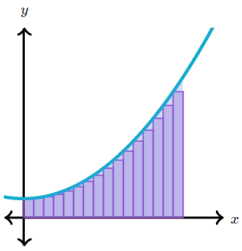


if we know the function f(x), for example: $f(x)= x^2+1$ *, then we can know the* ***height of each rectangle*** *and*

depending on how many rectangle we use, we will also know the ***width of each rectangle***

And our approximation gets better if we use more rectangles





An **Integral** *is created when we use calculus to add up an* ***infinite amount of rectangles***

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

You don't have to understand this formula yet, but you will when we discuss it in detail later. Many first year Calculus students get a tattoo of this (true story).

These sorts of approximations are called **Riemann sums**, and they're a foundational tool for integral calculus.

Integration Introduction Questions

1. What is the goal of **integration**?

2. Why is finding the area of under a function important?

3. Can you give some specific examples of applications for finding the area under a function?

4. Describe the technique that the integral uses to find areas under curvy functions. What is the name of this process called?

5. What does this symbol $\sum$ mean?

6. According to the formula on the previous page, which is used to help find the area under a curve a limit or a derivative?