Integration by parts and **u-substitution**
Integration (**Hand-in** Assignment)

By Parts: *Please complete on a separate sheet (show your work).*

$$\int f(x)\, g'(x) = f(x)\, g(x) - \int f'(x)\, g(x)$$

1. $\int 3x\ e^{-x}\ dx$
2. $\int \frac{\ln x}{x^2}\ dx$
3. $\int x^2\ \cos x\ dx$
4. $\int x\ \sin x\ \cos x\ dx$
5. $\int \cos^{-1} x\ dx$
6. $\int (\ln x)^2\ dx$
7. $\int x^3\ \sqrt{9-x^2}\ dx$
8. $\int e^{2x}\ \sin x\ dx$
9. $\int x^2\ \sqrt{x-1}\ dx$
10. $\int \frac{1}{x(\ln x)^3}\ dx$

U-Substitution: *Please complete on a separate sheet (show your work).*

1. $\int \sqrt{x-2}\,dx$

2. $\int (2x+3)^{11}\,dx$

3. $\int \sqrt{5x-1}\,dx$

4. $\int \sqrt[3]{6x+1}\,dx$

5. $\int 5(3-4x)^{2/3}\,dx$

6. $\int \frac{dx}{(8x-1)^3}$

7. $\int x(x^2+2)^6\,dx$

8. $\int 6x^2\sqrt{3x^3-1}\,dx$

9. $\int \left(1+\frac{1}{x}\right)^3\left(\frac{1}{x^2}\right)dx$

10. $\int x^{1/3}\left(x^{4/3}+9\right)^8\,dx$

11. $\frac{2}{3}\int \sqrt{4-\frac{3}{5}x}\,dx$

12. $\int (3x+15)\sqrt{x^2+10x+4}\,dx$

13. $\int (x+2)\sqrt{x-2}\,dx$

14. $\int \frac{x^2}{\sqrt{x-4}}\,dx$

15. $\int \sin 5x\,dx$

16. $\int \cos\frac{x}{2}\,dx$

17. $\int \frac{1}{3}\sec^2 8x\,dx$

18. $\int \sin 4x\cos 4x\,dx$

19. $\int \cos^3 x\sin x\,dx$

20. $\int \tan x\sec^2 x\,dx$

21. $\int \sqrt{\cos 6x}\sin 6x\,dx$

22. $\int \frac{\sin x}{(4-\cos x)^3}\,dx$

Integration by parts and **u-substitution** Integration (Hand-in Assignment)

Solutions

By Parts:

1. $-3xe^{-x} - 3e^{-x} + C$ $u = 3x$ $dv = e^{-x}dx$

2. $-\frac{\ln x}{x} - \frac{1}{x} + C$ $u = \ln x$ $dv = \frac{1}{x^2}dx$

3. $x^2 \sin x + 2x\cos x - 2\sin x + C$ $u = x^2$ $dv = \cos x \ \ dx$

4. $-\frac{x\cos 2x}{4} + \frac{\sin 2x}{8} + C$ note: $\frac{\sin 2x}{2} = \sin x \ \ \cos x$ $u = x$ $dv = \sin 2x \ \ \cos x \ \ dx$

5. $x \ \ \cos^{-1} x \ \ - \ \ \sqrt{1-x^2} + C$ $u = \cos^{-1} x$ $dv = dx$

6. $x(\ln x)^2 - 2x \ \ \ln x + 2x + C$ $u = (\ln x)^2$ $dv = dx$

7. $-\frac{x^2}{3}(9-x^2)^{3/2} - \frac{2}{15}(9-x^2)^{5/2} + C$ $u = x^2$ $dv = (4-x^2)^{1/2} \ \ x \ \ dx$

8. $\frac{2e^{2x}\sin x}{5} - \frac{e^{2x}\cos x}{5} + C$ $u = \sin x$ $dv = e^{2x}dx$

9. $\frac{2x^2(x-1)^{3/2}}{3} - \frac{8x(x-1)^{5/2}}{15} + \frac{16(x-1)^{7/2}}{105} + C$ $u = x^2$ $dv = (x-1)^{1/2}dx$

10. $\frac{-1}{2(\ln x)^2} + C$ $u = \frac{1}{(\ln x)^3} = (\ln x)^{-3}$ $dv = \frac{1}{x}dx$

U-Substitution:

1. $\int \sqrt{x-2}\,dx$

$$\frac{2(x-2)^{\frac{3}{2}}}{3}+C$$

2. $\int (2x+3)^{11}\,dx$

$$\frac{(2x+3)^{12}}{24}+C$$

3. $\int \sqrt{5x-1}\,dx$

$$\frac{2(5x-1)^{\frac{3}{2}}}{15}+C$$

4. $\int \sqrt[3]{6x+1}\,dx$

$$\frac{(6x+1)^{\frac{4}{3}}}{8}+C$$

5. $\int 5(3-4x)^{2/3}\,dx$

$$\frac{-3(3-4x)^{\frac{5}{3}}}{4}+C$$

6. $\int \frac{dx}{(8x-1)^3}$

$$\frac{-1}{16(8x-1)^2}+C$$

7. $\int x(x^2+2)^6\,dx$

$$\frac{(x^2+2)^7}{14}+C$$

8. $\int 6x^2\sqrt{3x^3-1}\,dx$

$$\frac{4(3x^3-1)^{\frac{3}{2}}}{9}+C$$

9. $\int \left(1+\frac{1}{x}\right)^3\left(\frac{1}{x^2}\right)dx$

$$\frac{-\left(1+\frac{1}{x}\right)^4}{4}+C$$

10. $\int x^{1/3}\left(x^{4/3}+9\right)^8\,dx$

$$\frac{\left(x^{4/3}+9\right)^4}{12}+C$$

11. $\frac{2}{3}\int \sqrt{4-\frac{3}{5}x}\,dx$

$$\frac{-20\left(4-\frac{3}{5}x\right)^{\frac{3}{2}}}{27}+C$$

12. $\int (3x+15)\sqrt{x^2+10x+4}\,dx$

$$(x^2+10x+4)^{\frac{3}{2}}+C$$

13. $\int (x+2)\sqrt{x-2}\,dx$

$$\frac{2(x-2)^{\frac{5}{2}}}{5}+\frac{8(x-2)^{\frac{3}{2}}}{3}+C$$

14. $\int \frac{x^2}{\sqrt{x-4}}\,dx$

$$\frac{2(x-4)^{\frac{5}{2}}}{5}+\frac{16(x-4)^{\frac{3}{2}}}{3}+32(x-4)^{\frac{1}{2}}+$$

15. $\int \sin 5x\,dx$

$$\frac{-\cos 5x}{5}+C$$

16. $\int \cos\frac{x}{2}\,dx$

$$2\sin\frac{x}{2}+C$$

17. $\int \frac{1}{3}\sec^2 8x\,dx$

$$\frac{\tan 8x}{24}+C$$

18. $\int \sin 4x\cos 4x\,dx$

$$\frac{\sin^2 4x}{8}+C \text{ or } \frac{-\cos^2 4x}{8}+C$$

19. $\int \cos^3 x\sin x\,dx$

$$\frac{-\cos^4 x}{4}+C$$

20. $\int \tan x\sec^2 x\,dx$

$$\frac{\tan^2 x}{2}+C$$

21. $\int \sqrt{\cos 6x}\sin 6x\,dx$

$$\frac{-\left(\cos(6x)\right)^{\frac{3}{2}}}{9}+C$$

22. $\int \frac{\sin x}{(4-\cos x)^3}\,dx$

$$\frac{-1}{2(4-\cos x)^2}+C$$