Note Title

DIFFERENTIAL EQUATIONS EXAMPLES.

6/1/2011

Ex. 1

$$\frac{dy}{dt} = \frac{4\sin(2t)}{y}$$

Always do the following:

$$y\,dy = 4\sin(2t)\,dt$$

a) seperate variables

$$\int y\,dy = \int 4\sin(2t)\,dt$$

b) Integrate both sides

$$\frac{1}{2}y^2 = \frac{-4\cos 2t}{2} + C$$

c) solve for y

$$y^2 = -4\cos 2t + 2c$$

$$y = \sqrt{2c - 4\cos 2t}$$ ☺✓

Ex #2

$$\frac{dy}{dt}(-yt) = 1 + 2t^2$$

a) seperate variables

$$-y\,dy = \frac{1 - 2t^2}{t}\,dt$$

$$-y\,dy = \left(\frac{1}{t} - 2t\right)dt$$

$$-\int y\,dy = \int\left(\frac{1}{t} - 2t\right)dt$$

b) Integrate both sides

$$-\frac{1}{2}y^2 = \ln t - t^2 + c$$

$$y^2 = -2\ln t + 2t^2 - 2c$$

c) solve for y

$$y = \sqrt{-2\ln t + 2t^2 - 2c}$$

Ex 3

$$\frac{dy}{dx}\,\frac{1}{y} = x$$

a) seperate variables

$$\frac{1}{y}\,dy = x\,dx$$

$$\int \frac{1}{y}\,dy = \int x\,dx$$

b) Integrate both sides

$$\ln|y| = \frac{1}{2}x^2 + C$$

c) solve for y

(TO GET RID OF ln

$$e^{\ln|y|} = e^{\frac{1}{2}x^2 + C}$$

$$|y| = e^{\frac{1}{2}x^2 + C}$$

$$y = \pm e^{\frac{1}{2}x^2 + C}$$

$$y = \pm e^{\frac{1}{2}x^2} \cdot e^{C}$$

$$y = Ae^{\frac{1}{2}x^2}$$

WE "e" BOTH SIDES

e IS INVERSE OF ln

Let $A = \pm e^{C}$

INITIAL CONDITION

$$\frac{dy}{dx} = 4x^3 y \qquad \text{FOR } f(1) = 1$$

$$\frac{1}{y}\,dy = 4x^3\,dx$$

a) seperate variables

$$\int \frac{1}{y}\,dy = \int 4x^3\,dx$$

b) Integrate both sides

c) solve for y

$$\ln|y| = x^4 + C$$

"e" BOTH SIDES

$$e^{\ln|y|} = e^{x^4 + C}$$

$$y = \pm e^{x^4 + C}$$

$$y = \pm e^{x^4} \cdot e^{C}$$

let $\pm e^{C} = A$

$$y = A e^{x^4}$$

SOLVE FOR A IF

$f(1) = 1$

$$1 = A e^{1}$$

$$A = \frac{1}{e}$$

$$y = \frac{e^{x^4}}{e}$$