*Derivative of **Exponential** Functions and **Log** Functions*

DERIVATIVES OF EXPONENTIAL FUNTIONS AND LOGRITHMIC FUNCTIONS

2014-03-05

HAVE YOU EVER SEEN THIS TYPE OF FUNCTION?

$y = 2^x$ ← EXPONENTIAL FUNCTION

THIS IS AN EXPONENTIAL FUNCTION AND IT HAS A SHAPE THAT MODELS ALL SORTS OF STUFF

EXPONENTIALS:

$$y = a^x$$
$$y' = a^x \ln a$$

or

$$y = a^{f(x)}$$
$$y' = a^{f(x)} \cdot f'(x) \ln a$$

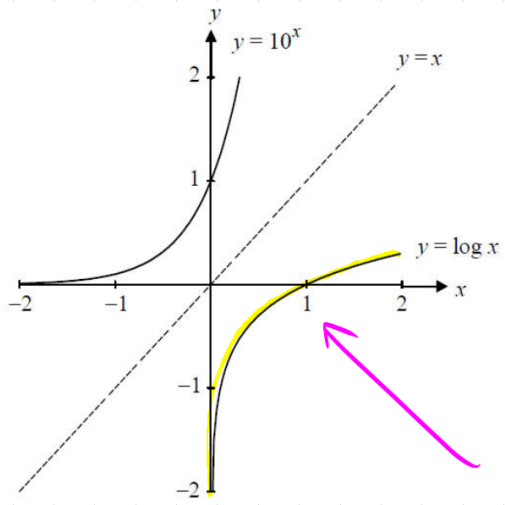
Examples:

$y = 5^x$

$y' = 5^x \ln 5$

$y = 5^{3x^2+2x}$

$y' = 5^{3x^2+2x} \cdot (6x+2) \ln 5$



EXPONENTIAL FUNCTIONS HAVE AN INVERSE TOO.

EX. $y = 2^x$ {SUB IN x TO GET y}

$14 = 2^x$ {GIVEN A y VALUE, HOW DO YOU GET x?}

HOW DO YOU "UNDO" $y = 2^x$ ⇒ LOGRITHMIC FUNCTION!

NOTE:

$$y = \log_a x$$

$$y' = \frac{1}{x} \cdot \frac{1}{\ln a}$$

$$y = \log_a f(x)$$

$$y' = \frac{1}{f(x)} \cdot f'(x) \cdot \frac{1}{\ln a}$$

Ex.

$$y = \log_5 x$$

$$y' = \frac{1}{x} \cdot \frac{1}{\ln 5}$$

$$y' = \frac{1}{x \ln 5}$$

Ex.

$$y = \log_5 x^2$$

$$y' = \frac{1}{x^2} \cdot 2x \cdot \frac{1}{\ln 5} = \frac{2}{x \ln 5}$$

Derivatives of Exponential and Logarithmic Functions

Find the derivatives of the following functions:

$y = 2^x$

$y = 43^{\sqrt{x}}$

$y = b^x$

$y = 7^{x^2}$

$y = 3^{\sin x}$

Answers:

$$\frac{dy}{dx} = 2^x \ \ln 2$$

$$\frac{dy}{dx} = \frac{\ln 43 \ 43^{\sqrt{x}}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = b^x \ln b$$

$$\frac{dy}{dx} = 2x \ 7^{x^2} \ln 7$$

$$\frac{dy}{dx} = 3^{\sin x} \cos x \ \ln 3$$

Find the derivatives of the following functions:

$$y = \log_2 \cos x.$$

$$y = \log_3 \frac{3}{x} + \frac{3}{x}.$$

$$f(x) = \log_{10}(x^3 + 1)$$

Answers:

$$y'(x)=\left(\log_2\cos x\right)'=\frac{1}{\cos x\cdot\ln 2}\cdot\left(\cos x\right)'=\frac{1}{\cos x\cdot\ln 2}\cdot\left(-\sin x\right)=-\frac{\sin x}{\cos x\cdot\ln 2}=-\frac{\tan x}{\ln 2}.$$

$$y'(x)=\left(\log_3\frac{3}{x}+\frac{3}{x}\right)'=\left(\log_3\frac{3}{x}\right)'+\left(\frac{3}{x}\right)'=\frac{1}{\frac{3}{x}\ln 3}\cdot\left(\frac{3}{x}\right)'+3\cdot\left(\frac{1}{x}\right)'=\frac{x}{3\ln 3}\cdot 3\cdot\left(-\frac{1}{x^2}\right)+3\cdot\left(-\frac{1}{x^2}\right)$$

$$=-\frac{3}{x^2}\left(\frac{x}{3\ln 3}+1\right)=-\frac{3}{x^2}\cdot\frac{x+3\ln 3}{3\ln 3}=-\frac{x+3\ln 3}{x^2\ln 3}.$$

$$f^{\backslash}(x)=\frac{3x^2}{x^3+1}\cdot\frac{1}{\ln 10}$$

Workbook:

Do pg. 166 1 to 14

Do pg. 172 6,9,10,11